

## The inconsistency of level critical thinking in solving differential equation problem

S S Faradiba<sup>1\*</sup>, P Andriani<sup>2</sup>, A Alifiani<sup>1</sup>, S E Walida<sup>1</sup>, D Daryono<sup>3</sup>, S N Hasana<sup>1</sup>,  
A D Angriani<sup>4</sup>, D Chamidah<sup>5</sup>, E Defitriani<sup>6</sup>, M T Qurohman<sup>7</sup>

<sup>1</sup>Jurusan Pendidikan Matematika, Universitas Islam Malang, Jl. M.T. Haryono No. 193, Malang, Indonesia

<sup>2</sup>Jurusan Tadris Matematika, Universitas Islam Negeri Mataram, Jl. Pendidikan No.35, Dasan Agung Baru, Selaparang, Mataram, Indonesia

<sup>3</sup>Jurusan Pendidikan Kewarganegaraan, STKIP PGRI Pasuruan, Jl. Ki Hajar Dewantara No. 27-29, Tembokrejo, Pasuruan, Indonesia

<sup>4</sup>Jurusan Pendidikan Matematika, Universitas Islam Negeri Alauddin, Jalan H. M. Yasin Limpo No. 36 Semata Gowa, Sulawesi Selatan, Indonesia

<sup>5</sup>Jurusan Pendidikan Biologi, Universitas Wijaya Kusuma, Jalan Dukuh Kupang XXV/54 Surabaya, Indonesia

<sup>6</sup>Jurusan Pendidikan Matematika, Universitas Batanghari, Jalan Slamet Riyadi Broni, Jambi, Indonesia

<sup>7</sup>Jurusan Teknik Mesin, Politeknik Harapan Bersama, Jalan Dewi Sartika 71 Tegal, Indonesia

\*[suryasarifaradiba@unisma.ac.id](mailto:suryasarifaradiba@unisma.ac.id)

**Abstract** Differential equation is a concept that has a variety of solutions that require critical thinking ability. This study explores the Level of Critical Thinking Ability (LCTA) of students in solving differential equation problem. This study is a descriptive qualitative research. The participants of this study were two university students of department of mathematics education, Universitas Islam Malang, who are taking differential equations course. The instruments were critical thinking questions based on *IDEAS* (Identify, Deepen, Enumerate, Assess, and Scrutinize) criteria and interview guideline. The results of the study showed that the level of critical thinking of students in solving the problem of differential equation is inconsistent. In this case, the inconsistency of the critical thinking level is caused by the obstacles that the students experience when solving the differential equation problem. The student who have problems in solving differential equations will trigger to think critically. It is expected that further research need to be carried out to examine the level of students' critical thinking in other courses.

### 1. Introduction

In the era of globalization, information can be accessed in so many ways, especially in the education community. Easy access to the internet has made the information available on the Internet unstructured and disorganized [1]. This condition requires each individual to develop high-level thinking skills, one of which is the ability to think critically. The ability to think critically is one of the four key skills students must possess [2].

Based on the literature review, several studies on critical thinking in solving general mathematical problems [3], algebra [4], statistics [5], geometry [6] have been conducted. However, research on critical thinking on the problem of differential equation is still limited. In fact, differential equation is courses can be applied in various fields, such as physics, engineering, and economics. In this case, the

differential equation bridges the problem in the real world with abstract mathematics. Therefore, the ability to think critically in analyzing the problems is needed in this course.

A differential equation is any equation which contains derivatives, either ordinary derivatives or partial derivatives. A differential equation is called an ordinary differential equation, if it has ordinary derivatives in it. Likewise, a differential equation is called a partial differential equation, if it has partial derivatives in it [7]. There are many possible conditions in solving the problem of differential equations: first, not all differential equations will have solutions so it's useful to know a head of time if there is a solution or not. Second, it is possible for a differential equation to have more than one solution. But, just because we know that a solution to differential equations exists does not mean that we will be able to find it. In fact, there is many ways to find a solution of differential equation. With regard to these conditions, critical thinking is needed in solving differential equations. The critical thinking is mental processing that organize and play a role in decision making process to resolve the problem [8].

Facione introduces five stages in critical thinking abbreviated as IDEAS (Identify, Deepen, Enumerate, Assess, and Scrutinize) [9]. Critical thinking process begins with identify the problem and set priorities. In the second steps, deepen understanding and gather relevant information. Next, critical thinking can do by enumerate options and anticipate consequences. Furthermore, assess the situation and make a preliminary decision. Finally, scrutinize the process and self-correct as needed to complete process of critical thinking.

Rasiman performs leveling of critical thinking in solving mathematical problems based on IDEAS [10]. Some cognitive abilities are described in each of the critical thinking stages, that is: Abilities of interpreting (Identify); Analyzing (Deepen); Evaluating (Enumerate); Concluding and explaining what they are thinking about, and making decision (Assess); and Applying the power of critical thinking to himself and improving the competence of he made (Scrutinize). A student who is able to do cognitive abilities above then his critical thinking ability is far above a person who is only able to interpret, analyze and evaluation. Thus it can be said that there is a leveling of critical thinking ability of a person. The level of critical thinking abilities of each person is different and these differences can be viewed as a continuum that starts from the lowest to the highest degree.

The levels of critical thinking by Rasiman are: students who are not critical (LCTA-0), students are less critical (LCTA-1), critical student (LCTA-2), and students are very critical (LCTA-3). This leveling is used to describe the students' critical thinking processes in this study. The purpose of this study are to describe the process of critical thinking in solving differential equations based on *IDEAS* and analyze to find whether the level of critical thinking process in students is consistent.

## 2. Method

This research was a qualitative research. The purpose of this research is to analyze the level of the students' critical thinking in solving differential equation problem according to IDEAS (Identify, Deepen, Enumerate, Assess, and Scrutinize) criteria. Furthermore, this research analyse the level of critical thinking and make sure that the levels are consistent This study involved of two students who got the highest grade in the differential equation middle test in Differential Equation Course, Department of Mathematics Education, Universitas Islam Malang.

Data collection techniques and analysis are explained as follows. The critical thinking test is based on critical thinking indicators. The course was taught by a lecturer other than the researchers. The interview in this research was conducted in three forms: structured, semi-structured, and unstructured interviews. Triangulation was done as a data collection technique that combines from various data collection techniques and data sources that already exist. The purpose of triangulation in this study was not to seek the truth about some phenomena, but rather to increase understanding of what was found.

In order to assess critical thinking ability, two differential equation problems were used.

*Solve differential equations below:*

1.  $(x + 2y)dx + (x + 2y)dy = 0$
2.  $(x + 2y + 3)dx + (2x + 3y + 4)dy = 0$

Those problems were selected because differential equation can be solved in so many ways depending on its types. The first differential equation can be categorized into three types: non separated differential equation that can be reduced, not exact differential equation, and homogenous differential equation.

Therefore, the first problem can be solved in three ways. Meanwhile, the second differential equation problem can be categorized into two types: exact differential equation and non-homogenous differential equation. This last type can be solved in two ways. Thus, before solving differential equations, the students must identify the type of differential equation at first. Then, they should determine the most effective way to find the solution.

### 3. Result and Discussion

This research combined IDEAS strategy and the level of critical thinking by Rasiman to analyse whether critical thinking ability of two students are always consistent in the first problem and the second problem [9,10]

**Table 1.** The level of critical thinking students

Steps	Characteristics	Level of Critical Thinking
<b>Identify (Step 1)</b> <b>Identifying the problem and setting priorities and Deepen (Step 2)</b> <b>Understanding and gathering relevant information</b>	<ol style="list-style-type: none"> <li>1. The students are less clear in identifying the facts in issue.</li> <li>2. The students are not precise and less clear in expressing the prerequisite knowledge (definition/ theorem/ data) that can be used in solving problems and in the end the students are not able to make plans based on knowledge preconditions problem solving.</li> <li>3. The students in solving problems based on concepts and ideas in the form of definitions, concepts, theorems, principles and procedures that are not clear, inaccurate, irrelevant and depth. Experiencing difficulty in implementing problem solving</li> <li>4. The students are vague and lacking in evaluating logical arguments used in solving the problem</li> </ol>	Non-critical students (LCTA-0)
<b>Enumerate (Step 3)</b> <b>Selecting and anticipating consequences</b>	<ol style="list-style-type: none"> <li>1. Students clearly identify the fact that there is a problem, well known facts or facts in question.</li> <li>2. Students are less precise and less clear in expressing the prerequisite knowledge (definition / theorem / data) to be used in solving the problem so that students are not exactly in a problem-solving plan based on prerequisite knowledge</li> <li>3. Students in resolving the problem has not been based on concepts and ideas in the form of definitions, concepts, theorems, principles. The procedure used to solve the problem too vague and imprecise.</li> <li>4. Students are vague and lacking in evaluating logical arguments used in examining the steps to solve the problem.</li> </ol>	Students are less critical (LCTA-1)
<b>Assess (Step 4)</b> <b>Assessing the situation and making a preliminary decision</b>	<ol style="list-style-type: none"> <li>1. Students identify clearly the fact that there is the problem. It can be seen from the students' ability to formulate the basic problem with the information known to the problem</li> <li>2. Students precise and clear in expressing the prerequisite knowledge (definition / theorem / data) that can be used in solving the problem, so that ultimately the student is able to make a plan solving the problem based on the facts given, prerequisite knowledge, clear procedures.</li> <li>3. Students can solve the problem but it is less clear in each stage are implemented, yet deep in the provision of</li> </ol>	Critical students (LCTA-2)

	argumentation steps undertaken and less profound in making modelling a given problem	
	4. The student has not been able to distinguish between conclusions based on valid logic.	
<b>Scrutinize (Step 5)</b> <b>Scrutinizing the process and self-correcting as needed</b>	<ol style="list-style-type: none"> <li>1. Students identify clearly the fact that there is the problem. It can be seen from the students' ability to formulate the basic problem with the known information in the problem and can provide a logical reason.</li> <li>2. Students precise and clear in expressing the prerequisite knowledge (definition / theorem / data) that can be used in solving the problem, and ultimately the students are able to make a plan solving the problem based on the facts given, prerequisite knowledge, clear procedures with stages logical</li> <li>3. Students in solving problems based on concepts and ideas in the form of definitions, concepts, theorems, principles and procedures that are clear, precise, relevant and insight full</li> <li>4. Students cannot distinguish between conclusions based on valid logic</li> </ol>	Students who are very critical (LCTA-3)

### 3.1 Analysis of Subject 1 (S1)

In the first problem, S1 identified the problem and found that the term was the same, that is  $(x+2y)$ . So it can be simplified with multiplying by the integration factor  $\frac{1}{(x+2y)}$ . In this case, S1 performed *identification*, *deepen*, *enumeration*, and *assess* simultaneously. Next, S1 performed *scrutinize* and found the writing error, the "+" replaced with "=".

$$\begin{aligned} (x+2y)dx + (x+2y)dy &= 0 \\ \hline \times \frac{1}{(x+2y)} \\ dx + dy &= 0 \\ \text{solusi :} \\ \int dx + \int dy &= C \\ x + y &= C \end{aligned}$$

**Figure 1.** The answer of S1 for the first problem

The same thing also happens in the second problem. S1 identified that the equation was not homogeneous. Furthermore, she assumed that the solution was too long, so S1 modified the form of differential equation by doing the transformation and solved it. So, S1 is categorized into the level of very critical thinking in the first and second problems.

Handwritten mathematical work showing the substitution method for a differential equation. The equations are:

$$(u + 2y + 3) du + (2u + 3y + 4) dy = 0$$

misal :  $u + 2y + 3 = u$

$$du + 2dy = du$$

$$du = du - 2dy$$

$$u (du - 2dy) + (2u - y - 2) dy = 0$$

**Figure 2.** The answer of S1 for the second problem

### 1.2 Analysis of Subject 2 (S2)

First, S2 solved the first problem by identifying the differential equation and categorizing it as a homogeneous differential equation. In this case, S2 identified the form of differential equation and obtained the two same terms, that is  $(x + 2y)$

Handwritten mathematical work showing the identification of a homogeneous differential equation. The equations are:

$$(x + 2y) dx + (x + 2y) dy = 0$$

PD homogen berderajat 1

**Figure 3.** Identify step of S2 when solving the first problem

Furthermore, S2 performed *deepen* to make sure that the differential equation is the homogeneous equation. In this case, S2 found no constraint when solving the second problem so as to solve the problem procedurally.

Handwritten mathematical work showing the substitution method for a homogeneous differential equation. The equations are:

$$(x + 2y) dx + (x + 2y) dy = 0$$

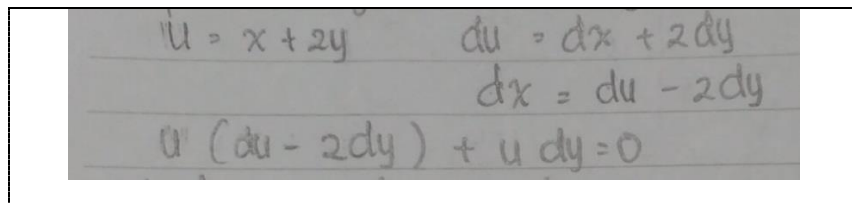
$$1 + 2\left(\frac{y}{x}\right) dx + 1 + 2\left(\frac{y}{x}\right) dy = 0$$

homogen.

**Figure 4.** Deepen step of S2 when solving first problem

However, S2 could not solve this differential equation by homogeneous differential equation method, so she considered another method, that is transformation and solve it. So, we conclude that S2 performed enumerate and assess simultaneously on that step. Finally, S2 performed scrutinize to make sure the solution was true. So, S2 can be categorized as very critical student (LCTA 3) in the first problem.





**Figure 5.** Transformation step of S2 when solving the first problem

For the second question, S2 only performed identify and deepen. Furthermore, S2 solved the second problem of differential equation procedurally without considering other ways. In this case, S2 was only able to identify the fact given clearly. S2 identified that the equation was not homogeneous so he deepened understanding and gathered relevant information about it to solve this problem. Based on this fact, S2 can be identified as not critical student (LCTA 0) when solving the second problem.

#### 4. Conclusion

The level of critical thinking of the students in solving the problem of differential equation is inconsistent. This study found that S1 has a high critical thinking level (LCTA 3) in both problems. Meanwhile, S2 has a high level of critical thinking (LCTA 3) only in the first problem. S2 can be categorized into first level of critical thinking (LCTA 0) in the second problem. In this case, the inconsistency of the critical thinking level is caused by the obstacles that the students experience when solving the differential equation problem. Students who have problems in solving differential equations will trigger to think critically. This research implies further research to investigate the level of students' critical thinking in other courses.

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